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M-theory superalgebra from multiple membranes

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Abstract: We investigate space-time supersymmetry of the model of multiple M2-branes proposed by Bagger-Lambert and Gustavsson. When there is a central element in Lie 3-algebra, the model possesses an extra symmetry shifting the fermions in the central element. Together with the original worldvolume supersymmetry transformation, we construct major part of the eleven dimensional space-time super-Poincaré algebra with central extensions. Implications to transverse five-branes in the matrix model for M-theory are also discussed.

Keywords: p-branes, Space-Time Symmetries, M-Theory.

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Contents

1. Introduction

Bagger, Lambert [1 − 3] and Gustavsson [4, 5] discovered an interesting model for multiple M2-branes (which we will call BLG model in the following) based on an algebraic structure called Lie 3-algebra. Since membranes are expected to be the fundamental building blocks of M-theory, it is intriguing to ask how much does the BLG model know about M-theory. Important information of M-theory is contained in the structure of the eleven dimensional space-time superalgebra, or "M-theory superalgebra" [6]. The BLG model is not space-time supersymmetric, at least manifestly. However, since fundamental membrane action is expected to have space-time supersymmetry, we may hope that the BLG model can be related to a gauge-fixed form of some manifestly space-time supersymmetric formulation.

In this paper we show that the most part of the eleven dimensional space-time super-Poincaré algebra with central extensions can actually be constructed from the BLG model, and indeed it captures important aspects of M-theory; namely charges of BPS branes. One of the crucial ingredients in constructing the space-time superalgebra is an existence of a central element in the Lie 3-algebra which the BLG model is based on. The shift of bosonic as well as fermionic fields in this central element are symmetries of the BLG model. The shift of the bosonic fields corresponds to translations in space-time, whereas the shift in the fermionic fields represents non-linearly realized part of the space-time super-Poincaré algebra.

Similar discussions on the worldvolume supersymmetry algebra of the BLG model which is identified with the linearly realized part of the space-time supersymmetry can be found in a recent paper [7]. We extend the results by including configurations which take values in non-trace elements (trace elements and non-trace elements are defined in section 2.1) and obtained more central charges which provide necessary pieces of the M-theory superalgebra. The algebra and the central charges which arise by including the fermionic shift symmetry are our new results. One of our main interests is on the charge of the five-brane constructed in [8, 9], and they are obtained only by including the fermionic shift symmetry in the algebra. Five-brane charges are of particular interests because in the matrix model for M-theory [10] transverse five-branes are not seen in the superalgebra [11].

Space-time superalgebra of a deformed BLG model without central extensions was constructed in [12]. Other aspects of BPS configurations for the worldvolume supersymmetry of the BLG model were studied in [13 – 15].

2. Space-time superalgebra from multiple membranes

2.1 The Bagger-Lambert-Gustavsson model

The Bagger-Lambert action which was proposed as a description for multiple M2-branes [2] (see also [1, 3 – 5]) has $\mathcal{N} = 8$ worldvolume supersymmetry. Furthermore, it has a novel gauge symmetry based on an algebraic structure called Lie 3-algebra [16]. For a linear space $\mathcal{V} = \sum_{a=1}^{\dim \mathcal{V}} v_a T^a; v_a \in \mathbb{C}$, Lie 3-algebra structure is defined by a multi-linear map which we call 3-bracket $[*, *, *]$: $\mathcal{V}^{\otimes 3} \to \mathcal{V}$ satisfying following properties:

1. Skew-symmetry:
$$[A_{\sigma(1)}, A_{\sigma(2)}, A_{\sigma(3)}] = (-1)^{|\sigma|}[A_1, A_2, A_3]. \tag{2.1}$$

2. Fundamental identity:
$$\begin{aligned}&[A_1, A_2, [B_1, B_2, B_3]]\\ &\qquad = [[A_1, A_2, B_1], B_2, B_3] + [B_1, [A_1, A_2, B_2], B_3] + [B_1, B_2, [A_1, A_2, B_3]].\end{aligned} \tag{2.2}$$

A linear space endowed with a Lie 3-algebra structure will be called Lie 3-algebra and typically denoted as $\mathcal{A}$ in this paper. In terms of the basis T^a, Lie 3-algebra can be expressed in terms of the structure constants $f^{abc}{}_d$:

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d. \tag{2.3}$$

An element $T^a \in \mathcal{A}$ is called a center if $[T^a, T^b, T^c] = 0, \forall T^b, T^c \in \mathcal{A}$, and $f^{abc}{}_d = 0$ in this case. To construct the action, we will also need an inner product in Lie 3-algebra. We assume the structure $\mathcal{V} = \mathcal{V}_{tr} \oplus \mathcal{V}_{ntr}$, where elements in $\mathcal{V}_{tr}$ have inner product and elements in $\mathcal{V}_{ntr}$ do not. We will refer to the elements in $\mathcal{V}_{tr}$ as trace elements, and elements in $\mathcal{V}_{ntr}$

as non-trace elements. By definition, elements $T^a, T^b \in \mathcal{V}_{tr}$ have inner product $\langle *, * \rangle$: $\mathcal{V}_{tr} \otimes \mathcal{V}_{tr} \to \mathbb{C}$:

$$\langle T^a, T^b \rangle = h^{ab}. \tag{2.4}$$

We will call h^{ab} as metric of Lie 3-algebra. We require following invariance of the inner product which is important for the gauge invariance of the Bagger-Lambert action:

$$\langle [T^a, T^b, T^c], T^d \rangle + \langle T^c, [T^a, T^b, T^d] \rangle = 0. \tag{2.5}$$

Together with the skew-symmetry property (2.1), the invariance of the metric (2.5) requires the indices of structure constants $f^{abcd} \equiv f^{abc}{}_e h^{ed}$ to be totally anti-symmetric:

$$f^{abcd} = \frac{1}{4!} f^{[abcd]}. \tag{2.6}$$

Remember that (2.6) is guaranteed only for trace elements with invariant metric. Inner product and metric are not defined for non-trace elements. Nevertheless, the 3-bracket can map non-trace elements to a trace element. These non-trace elements will play important role in this paper. For more about Lie 3-algebra in the BLG model, see e.g. [17–25].

The Bagger-Lambert action is given by [2]

$$S = \int d^3x \, \mathcal{L}, \tag{2.7}$$

where the Lagrangian density $\mathcal{L}$ is given by

$$\mathcal{L} = -\frac{1}{2} \langle D^\mu X^I, D_\mu X^I \rangle + \frac{i}{2} \langle \bar{\Psi}, \Gamma^\mu D_\mu \Psi \rangle + \frac{i}{4} \langle \bar{\Psi}, \Gamma_{IJ} [X^I, X^J, \Psi] \rangle - V(X) + \mathcal{L}_{CS}. \tag{2.8}$$

$X^I \in \mathcal{V}_{tr}$[1] is a scalar field on the worldvolume and I is a SO(8) vector index. $\Psi \in \mathcal{V}_{tr}$ are Majorana spinors on $1+2$ dimensional worldvolume, but can be combined into a single Majorana spinor in eleven dimensions subject to the chirality condition $\Gamma \Psi = -\Psi$, $\Gamma \equiv \Gamma_{012}$. Notations for gamma matrices are summarized in the appendix. D_μ is the covariant derivative

$$(D_\mu X^I(x))_a = \partial_\mu X^I_a(x) - \tilde{A}_\mu{}^b{}_a(x) X^I_b(x), \quad \tilde{A}_\mu{}^b{}_a \equiv A_{\mu cd} f^{cdb}{}_a, \tag{2.9}$$

where A_μ is a worldvolume gauge field. $V(X)$ is the potential

$$V(X) = \frac{1}{12} \langle [X^I, X^J, X^K], [X^I, X^J, X^K] \rangle. \tag{2.10}$$

The Chern-Simons term for the gauge potential is given by

$$\mathcal{L}_{CS} = \frac{1}{2} \varepsilon^{\mu\nu\lambda} \left(f^{abcd} A_{\mu ab} \partial_\nu A_{\lambda cd} + \frac{2}{3} f^{cda}{}_g f^{efgb} A_{\mu ab} A_{\nu cd} A_{\lambda ef} \right). \tag{2.11}$$

[1] Later we will relax this condition slightly and allow constant backgrounds X^I to take values in non-trace elements.

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The Bagger-Lambert action is invariant under the following gauge transformation:

$$\begin{aligned}
\delta_\Lambda X^I_a &= \Lambda_{cd}[T^c, T^d, X^I]_a = \Lambda_{cd} f^{cde}{}_a X^I_e = \tilde{\Lambda}^e{}_a X^I_e, \\
\delta_\Lambda \Psi_a &= \Lambda_{cd}[T^c, T^d, \Psi]_a = \Lambda_{cd} f^{cde}{}_a \Psi_e = \tilde{\Lambda}^e{}_a \Psi_e, \\
\delta_\Lambda \tilde{A}_\mu{}^b{}_a &= \partial_\mu \tilde{\Lambda}^b{}_a - \tilde{\Lambda}^b{}_c \tilde{A}_\mu{}^c{}_a + \tilde{A}_\mu{}^b{}_c \tilde{\Lambda}^c{}_a, \quad \tilde{\Lambda}^b{}_a \equiv f^{cdb}{}_a \Lambda_{cd}.
\end{aligned} \tag{2.12}$$

The fundamental identity (2.2) leads to

$$\delta_\Lambda[\Phi(1), \Phi(2), \Phi(3)] = \Lambda_{cd}[T^c, T^d, [\Phi(1), \Phi(2), \Phi(3)]], \tag{2.13}$$

where Φ's collectively represent X^I and Ψ. The metric is not involved in reaching (2.13) and this formula applies to both trace elements and non-trace elements. On the other hand, the invariance of the metric (2.5) leads to

$$\delta_\Lambda \langle Y, Z\rangle = \Lambda_{ab}\left(\langle [T^a, T^b, Y], Z\rangle + \langle Y, [T^a, T^b, Z]\rangle\right) = 0. \tag{2.14}$$

for any trace elements Y, Z which transform as $\delta_\Lambda Y = \Lambda_{cd}[T^c, T^d, Y]$, $\delta_\Lambda Z = \Lambda_{cd}[T^c, T^d, Z]$. (2.13) and (2.14) can be used to show the gauge invariance of the Bagger-Lambert action.

2.2 Worldvolume supersymmetry of the BLG model

The Bagger-Lambert action is invariant under the following supersymmetry transformations:[2]

$$\begin{aligned}
\delta_\epsilon X^I_a &= i\bar{\epsilon}\Gamma^I \Psi_a, \\
\delta_\epsilon \Psi_a &= D_\mu X^I_a \Gamma^\mu \Gamma^I \epsilon - \frac{1}{6} X^I_b X^J_c X^K_d f^{bcd}{}_a \Gamma^{IJK}\epsilon, \\
\delta_\epsilon \tilde{A}_\mu{}^b{}_a &= i\bar{\epsilon}\Gamma_\mu \Gamma_I X^I_c \Psi_d f^{cdb}{}_a,
\end{aligned} \tag{2.15}$$

where the supersymmetry parameter satisfies $\Gamma\epsilon = \epsilon$. The charge density, i.e. the temporal component of the Noether current associated with the supersymmetry transformation (2.15), is found to be

$$q^L = -\Gamma^\mu \Gamma^I \Gamma^0 \langle D_\mu X^I, \Psi\rangle - \frac{1}{6}\Gamma^{IJK}\Gamma^0 \langle [X^I, X^J, X^K], \Psi\rangle, \tag{2.16}$$

and the Noether charge is

$$Q^L = \int d^2x\, q^L. \tag{2.17}$$

The suffix L indicates that it is identified with the linearly realized part of the space-time supersymmetry.

In this paper we will often be interested in central charges which are proportional to the volume of the membranes, which can be infinite for infinitely extended membranes. A standard way to avoid infinities associated with such infinite volume arising from the (anti-)

[2]See [26] for a $\mathcal{N} = 1$ superfield formalism.

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commutation relations of Noether charges is to use charge density. In the following, it is understood that the fermions Ψ are set to zero after calculating the Dirac bracket, since we are interested in bosonic backgrounds. The Dirac bracket of q^L and Q^L is calculated to be

$$\begin{aligned} i\{q^L, Q^L\}_D = 2p_\mu\Gamma_+\Gamma^\mu C \\ +z_{IJ}\Gamma^{IJ}C + z_{0ijIJ}\Gamma^{0ijIJ}C \\ +z_{0iIJKL}\Gamma^{0iIJKL}C + z_{jIJKL}\Gamma^{jIJKL}C \\ +z_{0IJKL}\Gamma^{0IJKL}C + z_{ijIJKL}\Gamma^{ijIJKL}C, \end{aligned} \tag{2.18}$$

where

$$z_{IJ} = \frac{1}{2}\left(-\varepsilon^{0ij}\langle D_iX^I, D_jX^J\rangle + \langle D_0X^K, [X^K, X^I, X^J]\rangle\right), \tag{2.19}$$

$$z_{0ijIJ} = \frac{1}{2}\left(\langle D_iX^I, D_jX^J\rangle - \frac{1}{2}\varepsilon^0{}_{ij}\langle D_0X^K, [X^K, X^I, X^J]\rangle\right), \tag{2.20}$$

$$z_{0iIJKL} = \frac{1}{6}\langle D_iX^I, [X^J, X^K, X^L]\rangle, \tag{2.21}$$

$$z_{iIJKL} = -\frac{1}{6}\varepsilon^{0j}{}_i\langle D_jX^I, [X^J, X^K, X^L]\rangle, \tag{2.22}$$

$$z_{0IJKL} = -\frac{1}{8}\langle [X^M, X^I, X^J], [X^M, X^K, X^L]\rangle, \tag{2.23}$$

$$z_{ijIJKL} = -\frac{1}{16}\varepsilon^0{}_{ij}\langle [X^M, X^I, X^J], [X^M, X^K, X^L]\rangle. \tag{2.24}$$

In the above, anti-symmetrization of the space-time indices is understood. And $\Gamma_\pm \equiv (1 \pm \Gamma)/2$. This projection arises from the chirality of the supercharges: $\Gamma Q^L = Q^L$. In the second, the third and the fourth lines of (2.18), two terms in the same line arise from two different Gamma matrices in the projection $\Gamma_+ = (1+\Gamma)/2$. The bosonic part of the Hamiltonian density is given by

$$\mathcal{H} = p_0 = \frac{1}{2}\langle D_0X^I, D_0X^I\rangle + \frac{1}{2}\langle D_iX^I, D_iX^I\rangle + V(X), \tag{2.25}$$

and the momentum density is given by

$$p_i = \langle D_0X^I, D_iX^I\rangle. \tag{2.26}$$

We refer to the appendix for details. These central charges have been discussed in [7];[3] the combination of the central charges (2.19) and (2.20) was found to be the charge of vortices, and identified with M2-branes intersecting with the multiple M2-branes. The combination of the central charges (2.21) and (2.22) was found to be the charge of Basu-Harvey solution [27] which had been identified with M2-branes ending on M5-branes. Readers interested in further discussions are advised to consult [7].

The central charges (2.23) and (2.24) vanish if we only consider trace elements in the Lie 3-algebra due to the total anti-symmetry of the indices I, J, K, L and the fundamental

[3]The expressions for the central charges look different just because we haven't used the equation of motions.

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identity (2.2). However, we would like to take into account constant background configurations of X^I's which take values in non-trace elements. As long as they give trace elements after putting into the 3-brackets in the Bagger-Lambert action, the action is still well-defined and gauge invariant, provided the fluctuations around the background are still restricted to trace elements.[4] This kind of configurations give rise to BPS brane charges. For example, in the case when the Bagger-Lambert action reduces to D2-brane action by giving expectation value to the field X_0^{10} in the notation of [22], (2.23) and (2.24) reduce to the form $\sim \mathrm{tr}[X^I, X^J][X^K, X^L]$, where $[*, *]$ is the commutator of matrices and tr is the trace for matrices, and the matrix size is to be taken to infinity.[5] This term is analogous to the D4-brane charge (as well as the charges of the D0-branes within the D4-branes) in the matrix model for M-theory found in [11], and one should keep this kind of terms in order to obtain all the BPS-brane charges in the model. In the current example, the action reduces to D2-branes instead of D0-branes for the matrix model, so the charge should be interpreted as D6-brane charge. This type of configuration is also crucial for the construction of the five-brane from the BLG model in [8, 9] and we will discuss this in section 2.4.

2.3 Space-time superalgebra from the BLG model

It has been noticed that the choice of Lie 3-algebra in the BLG model already contains the choice of space-time in which membranes are embedded [28 – 30]. This is not surprising if we recall the analogous situation in multiple D-brane worldvolume theory, where the gauge group contains information of space-time, e.g. orientifold for gauge group SO. In the BLG model, when there is a central element in the Lie 3-algebra there is a bosonic shift symmetry in this direction:

$$\begin{aligned} \delta_{\vec{a}} X^I_\odot &= a^I \quad (a^I : \text{constant}), \\ \delta_{\vec{a}} \Psi_a &= \delta_{\vec{a}} \tilde{A}_\mu{}^b{}_a = 0, \end{aligned} \tag{2.27}$$

as well as the fermionic shift symmetry[6]

$$\begin{aligned} \delta_\eta \Psi_a &= \delta_{a\odot} \eta, \\ \delta_\eta X^I_a &= \delta_\eta \tilde{A}_\mu{}^b{}_a = 0. \end{aligned} \tag{2.28}$$

We use index $\odot$ to denote the generator corresponding to the central element. In the following, we restrict ourselves to the case where the metric for this central element takes following form:

$$h^{\odot a} = \delta^{\odot a}. \tag{2.29}$$

[4]Recall (2.13) and (2.14). The configuration is gauge covariant, but the value of the action is invariant under gauge transformations with parameters taking values in trace elements.

[5]In this case, actually the commutator of X^I's are still non-trace elements, and the central charge diverges. This is attributed to the infinite volume of indefinitely extended D6-branes discussed below. The charge density per D6-brane worldvolume is still finite.

[6]The fermionic shift symmetry has been used in [9] to obtain the worldvolume supersymmetry of the five-brane action constructed from the BLG model.

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With this metric, it is natural to identify $X^I_\odot$ as the center of mass coordinate in the direction transverse to membranes up to normalization, and (2.27) is nothing but the translational symmetry in this direction. We further assume that there is only one such central element with metric of the form (2.29) in the Lie 3-algebra,[7] because it is strange if there are two sets of center of mass coordinates.[8] In our setting, the Noether charge density associated with the transformation (2.28) is given by

$$q^{NL} = -\Gamma^0 \Psi_\odot, \tag{2.30}$$

and the Noether charge is

$$Q^{NL} = \int d^2x\, q^{NL}, \tag{2.31}$$

where the suffix NL indicates that it is identified with the non-linearly realized part of the space-time supersymmetry. Note that Q^{NL} has the same chirality with the worldvolume fermions Ψ, i.e. $\Gamma Q^{NL} = -Q^{NL}$, as opposed to Q^L.

The BLG model is not space-time super-Poincaré symmetric, at least manifestly. However, if we want to regard the model as a description of multiple M2-branes, it is natural to expect that it is a gauge fixed form of some space-time supersymmetric and worldvolume reparametrization invariant formulation. In the case of single supermembrane, it has been shown that the space-time supersymmetry reduces to the worldvolume supersymmetry by static gauge fixing [31 – 33]. After the gauge fixing, the linearly realized part of the space-time supersymmetry becomes global supersymmetry on the worldvolume theory, whereas the Nambu-Goldstone modes for the non-linearly realized part of the space-time supersymmetry become fermion fields on the worldvolume [34]. In our case, fields Ψ can be thought of as Nambu-Goldstone fermions for non-linearly realized space-time supersymmetry. In the following we will show that the charge Q^{NL} associated with the fermionic shift symmetry (2.28) almost provides the non-linearly realized part of the space-time supersymmetry, though there is a missing piece as we will see shortly.

The Dirac bracket of q^{NL} and Q^L are given as

$$i\{q^{NL}, Q^L\}_D + i\{q^L, Q^{NL}\}_D = p_I \Gamma^I C + \frac{1}{2} z_{iI} \Gamma^{iI} C + \frac{1}{2} z_{ijIJK} \Gamma^{ijIJK} C, \tag{2.32}$$

where

$$p_I \equiv \partial_0 X^I_\odot \tag{2.33}$$

is the momentum density in the direction transverse to the membranes. The central charge densities are found to be

$$z_{iI} = 2\partial_j X^I_\odot \varepsilon_i{}^{j0}\,, \tag{2.34}$$

$$z_{ijIJK} = -\frac{1}{6} \varepsilon^0{}_{ij} \langle [X^I, X^J, X^K], T_\odot \rangle. \tag{2.35}$$

[7] We allow other central elements with non-positive-definite metric [20 – 22].
[8] Though it may work with some kind of gauging.

The central charge density (2.34) describes tilting multiple membranes. For example, let us consider the situation where $X^I_\odot$ is compactified on a circle with radius R^I, and x^j is compactified on a circle with radius r^j. Then

$$X^I_\odot = \frac{n}{m}\frac{R^I}{r^j}x^j \tag{2.36}$$

is a configuration of membranes which winds the I-th direction for n times and j-th direction for m times. This configuration gives topological winding numbers through the central charge density (2.34).

The central charge density (2.35) vanishes when all X^I's in the 3-bracket take values in trace elements of the Lie 3-algebra, due to the definition of the central element and the invariance of the metric (2.5). This is not necessarily the case if we consider constant configurations where X^I's take values in non-trace elements. As long as we obtain a trace element after putting such X^I's into the 3-bracket, the inner product in (2.35) is well-defined and gives a finite number. The Bagger-Lambert action is also well-defined for such configuration, provided it is regarded as a background;[9] fluctuations from the background should still be in the space of trace elements. To describe a five-brane in the BLG model based on Nambu-Poisson bracket [8, 9], the background configuration is indeed given by such X^I's taking values in non-trace elements, and (2.35) gives the charge of the five-brane. We will come back to this point again in section 2.4.

The Dirac bracket of q^{NL} and Q^{NL} is given by

$$i\{q^{NL}, Q^{NL}\}_D = \Gamma_-\Gamma^0 C = \frac{1-\Gamma}{2}\Gamma^0 C. \tag{2.37}$$

The last expression in (2.37) can be interpreted as a sum of the mass density and the charge density of the static multiple membranes. (The absence of such term in (2.18) can be regarded as cancellation of mass and charge for the BPS configuration of membranes [35].) However, it does not contain contributions from excitations on the worldvolume to the energy nor the momentum in the worldvolume direction, which are required for making up the eleven dimensional super-Poincaré algebra. Besides this point, we can construct full space-time supercharge density q and charge Q as follows:

$$q = q^L + 2\sqrt{N}q^{NL}, \tag{2.38}$$
$$Q = Q^L + 2\sqrt{N}Q^{NL}. \tag{2.39}$$

Here, we have introduced a constant N which can be interpreted as "number" of membranes. The reason for this factor is as follows: Up to normalization $X^I_\odot$ is interpreted as the center of mass coordinate in the transverse directions. To define the center of mass, we need to know the number of membranes. However, there's no definite rule for relating the dimension of a Lie 3-algebra and the number of membranes. In the case of the Lie 3-algebra constructed from ordinary Lie algebra in order to derive D2-brane action from the Bagger-Lambert action [20 − 22], the number of membranes should be equal to the

[9] Note that the covariant derivative (2.9) can be rewritten as $D_\mu X^I = \partial_\mu X^I - A_{\mu\, cd}[T^c, T^d, X^I]$.

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number of D2-branes and determined from the rank of the Lie group, e.g. N for $\mathrm{U}(N)$. We will discuss the case when Nambu-Poisson bracket is chosen as Lie 3-algebra in the next subsection. Since the number of membranes is decided case by case depending on the choice of Lie 3-algebra, we just denote this number as N.

Finally, we obtain the eleven dimensional super-Poincaré algebra with central extensions:

$$
\begin{aligned}
i\{q,Q\}_D = \; & 2(\Gamma^0-\Gamma^{12})CN + 2p_\mu\Gamma_+\Gamma^\mu C + 2p_I\Gamma^I C\sqrt{N} \\
& + z_{iI}\Gamma^{iI}C\sqrt{N} + z_{ijIJK}\Gamma^{ijIJK}C\sqrt{N} \\
& + z_{IJ}\Gamma^{IJ}C + z_{0ijIJ}\Gamma^{0ijIJ}C \\
& + z_{0iIJKL}\Gamma^{0iIJKL}C + z_{jIJKL}\Gamma^{jIJKL}C \\
& + z_{0IJKL}\Gamma^{0IJKL}C + z_{ijIJKL}\Gamma^{ijIJKL}C.
\end{aligned} \tag{2.40}
$$

As mentioned above, the first term of (2.40) is interpreted as coming form tension and charge per volume of N membranes. From the kinetic term the relative normalization between $X^I_\odot$ and the center of mass coordinate is read off as $X^I_\odot = \sqrt{N}X^I_{C.M.}$, where $X^I_{C.M.}$ is the center of mass coordinate. Then $p_I\sqrt{N} = p_I^{C.M.}N$ is the total momentum in the direction transverse to membranes, and N appears in an appropriate way for a number of membranes.

Eq. (2.40) is almost the eleven dimensional super-Poincaré algebra, except that the piece $2p_\mu\Gamma_-\Gamma^\mu C$ is missing in the right hand side of (2.40). It is important that the piece $2p_I\Gamma^I C$ for the eleven dimensional super-Poincaré algebra has been obtained. If we had a space-time supersymmetric formulation with worldvolume reparametrization invariance for multiple membrane action which reduces to the Bagger-Lambert action after gauge fixing, this would be understood as due to the static gauge and kappa symmetry gauge fixing. We hope to clarify this point in the future. Further speculations will be given in the last discussion section.

2.4 On M5-brane charges in the BLG model

An example of Lie 3-algebra is given by Nambu-Poisson bracket on an "internal" three-manifold. For simplicity, we take T^3 to be the internal three-manifold. For more about the use of Nambu-Poisson bracket in the BLG model, see [17, 8, 9]. We consider the Nambu-Poisson bracket on T^3 given by

$$
\{f,g,h\}_{\mathrm{NP}} = \sum_{\dot\mu\dot\nu\dot\lambda} \Omega\varepsilon^{\dot\mu\dot\nu\dot\lambda}\partial_{\dot\mu}f(y)\partial_{\dot\nu}g(y)\partial_{\dot\lambda}h(y). \tag{2.41}
$$

Here $y^{\dot\mu}$ ($\dot\mu = \dot1,\dot2,\dot3$) are flat coordinates on T^3 with the identification $y^{\dot\mu} \sim y^{\dot\mu} + 2\pi$, and Ω is a constant. The invariant inner product can be defined by the integral over T^3:

$$
\langle f,g\rangle \equiv \int_{T^3} d^3y\, f(y)g(y). \tag{2.42}
$$

The trace elements of the Lie 3-algebra are given by square-integrable periodic functions on T^3. If we denote the basis of such functions on T^3 as $\chi^a(y)$ ($a = 1,2,3,\cdots$), the

Nambu-Poisson bracket can be written with structure constants:

$$\{\chi^a, \chi^b, \chi^c\}_{\rm NP} = \sum_d f^{abc}{}_d \chi^d \,. \tag{2.43}$$

Using the definition of the Nambu-Poisson bracket (2.41), it is easy to check that the fundamental identity (2.2) holds. We normalize the basis as $\langle \chi^a, \chi^b \rangle = \delta^{ab}$; then the normalized central element is given as $T^{\odot} = 1/\sqrt{(2\pi)^3}$.

We would like to consider the case where the target space is also compactified on a T^3. By this we mean the identification in the central element:

$$X^I(y) \sim X^I(y) + 2\pi R^I, \tag{2.44}$$

for say $I = 3, 4, 5$, where R^I is the compactification radius in the I-th direction.

Now let us consider a background configuration

$$X^I = R^I m_I y^{\dot\mu}, \quad \dot\mu = I - 2 \quad (I = 3, 4, 5). \tag{2.45}$$

The functions $y^{\dot\mu}$ $(\dot\mu = \dot1, \dot2, \dot3)$ are not periodic functions on T^3: They have a jump at $y^{\dot\mu} = 2\pi$. However, when the target space is also compactified as in (2.44), such jump can be set to null for the configuration (2.45) due to the identification in the target space. In this case, it is natural to include these elements in the Lie 3-algebra. However, there is no natural way to define invariant inner product for these elements. For example,

$$\begin{aligned} &\int_{T^3} d^3y\, \{y^{\dot1}, y^{\dot2}, 1\}_{NP} \cdot y^{\dot3} = 0 \\ \neq - &\int_{T^3} d^3y\, 1 \cdot \{y^{\dot1}, y^{\dot2}, y^{\dot3}\}_{NP} = \Omega(2\pi)^3. \end{aligned} \tag{2.46}$$

This means that the integration over T^3 does not provide an invariant metric for these new elements. Therefore, these elements should be included as non-trace elements. In the Bagger-Lambert action, these X^I's in non-trace elements always appear inside the Nambu-Poisson brackets; and the Nambu-Poisson brackets with such non-trace elements give trace elements, since the derivative inside the Nambu-Poisson bracket acting on $y^{\dot\mu}$ gives a constant which is a trace element. As long as such configuration is regarded as a non-dynamical background independent of the worldvolume coordinates, the Bagger-Lambert action is still well-defined and gauge invariant.

Now we come back to the issue of the "number of membranes" discussed in the previous subsection. In the current case where there is a natural notion of identity "1" in the elements and the metric is positive definite, it is natural to interpret the number we get when we put "1" into the inner product as the number of membranes. This is nothing but the volume of the internal manifold T^3. Therefore we set $N = (2\pi)^3$.

The background configuration (2.45) contributes to the five-brane charge (2.35) as

$$z_{ijIJK}\sqrt{N} = -\frac{1}{3!}\varepsilon_{IJK}(2\pi)^3 \varepsilon^0{}_{ij} \Omega R^3 R^4 R^5 m_3 m_4 m_5. \tag{2.47}$$

(2.47) is interpreted as a charge of a five-brane wrapping the I-th direction for m_I times.

Note that the potential term in the Bagger-Lambert action can be rewritten as

$$\begin{aligned} V(X) &= \frac{1}{12}\Big(\langle [X^I, X^J, X^K] - W^{IJK}T^{\odot}, [X^I, X^J, X^K] - W^{IJK}T^{\odot}\rangle \\ &\qquad +2W^{IJK}\langle [X^I, X^J, X^K], T^{\odot}\rangle - W^{IJK}W^{IJK}\Big) \\ &= \frac{1}{12}\langle [X^I, X^J, X^K] - W^{IJK}T^{\odot}, [X^I, X^J, X^K] - W^{IJK}T^{\odot}\rangle \\ &\qquad -\frac{1}{2}W^{IJK}\varepsilon^{0ij}z_{ijIJK} - \frac{1}{12}W^{IJK}W^{IJK}, \end{aligned} \tag{2.48}$$

where W^{IJK} is a constant totally anti-symmetric tensor

$$W^{IJK} = \varepsilon^{IJK}\Omega R^3 R^4 R^5 m_3 m_4 m_5. \tag{2.49}$$

Therefore the static configuration (2.45) saturates the minimal energy bound for given winding numbers.

Some time ago a matrix model was proposed as a description of M-theory [10], and BPS branes in this model were analyzed from the central extension of the superalgebra [11]. It was found that the charge of transverse five-branes, i.e. five-branes transverse to the M-theory circle which relates M-theory to type IIA string, is absent in this model. This can be a problem if the model is the fundamental definition of M-theory, though the model may better be regarded as M-theory in a particular frame in which some information of the full M-theory has been dropped off. From our results for the M-theory superalgebra (2.40), we can draw a scenario for how such thing can happen in the BLG model: The action for the matrix model for M-theory is basically that of the large number of multiple D0-branes in type IIA string theory. From the Bagger-Lambert action, such action may be obtained by first reducing it to multiple D2-brane action [36, 20 – 22], then wrapping D2-branes on T^2, and then performing T-duality transformations in the T^2 directions. To obtain multiple D2-brane action from the Bagger-Lambert action, it is necessary to reduce Lie 3-algebra to ordinary Lie algebra. This should be achieved by some background configuration in the BLG model which describes a compactification on the M-theory circle. However, by this background configuration the five-brane charges expressed using Lie 3-algebra in (2.21), (2.22) or (2.35) must also reduce to the expression using ordinary Lie algebra. This should be interpreted as five-branes are also wrapping the circle direction. Thus when one obtains the matrix model for M-theory from the Bagger-Lambert action, transverse five-brane charges which uses Lie 3-algebra structure in an essential way, i.e. those which do not reduce to a form written with ordinary Lie algebra, necessarily drop out from the model.

3. Summary and discussions

In this paper we studied the space-time supersymmetry of the BLG model when there is a central element in the Lie 3-algebra, and obtained the eleven dimensional super-Poincaré algebra with central extensions, except the piece $2p_\mu\Gamma_-\Gamma^\mu C$. The first crucial ingredient in the construction of the space-time superalgebra was to include the fermionic shift symmetry associated with the central element in the Lie 3-algebra. This fermionic shift symmetry was identified with the non-linearly realized part of the space-time supersymmetry. Together with the linearly realized worldvolume supersymmetry, it makes up the eleven dimensional super-Poincaré algebra. The second important ingredient was to take into account the non-trace elements for constant background configurations. The central charges constructed from non-trace elements provide important pieces of the M-theory superalgebra. For example, the charge of the five-brane constructed in [8, 9] can only be constructed by taking into account such non-trace elements.

Compared with the matrix model for M-theory which can be regarded as regularization of supermembrane action in the light-cone gauge [37], the BLG model lacks relation to a manifestly space-time supersymmetric formulation at this moment. Nevertheless, in this paper we could obtain the eleven dimensional super-Poincaré algebra almost fully. This suggests the existence of a manifestly space-time supersymmetric formulation with worldvolume reparametrization invariance which reduces to the BLG model after gauge fixing. It will be very interesting to construct such manifestly space-time supersymmetric formulation for the BLG model, and understand why the piece $2p_\mu\Gamma_-\Gamma^\mu C$ is missing in our algebra. In the case where the Lie 3-algebra is Nambu-Poisson bracket, it is likely that such manifestly space-time supersymmetric formulation is some covariant formulation of single M5-brane worldvolume action in three-form field background rather than multiple M2-brane action: If we can find a way to relate such formulation to the five-brane action constructed from the Bagger-Lambert action in [8, 9], we will be able to understand the origin of our super-Poincaré algebra. An interesting worldvolume reparametrization invariant formulation of single M5-brane action which might be related to the BLG model was constructed in [38], though only the bosonic part has been worked out. A worldvolume supergravity action which in a limit reduces to the Bagger-Lambert action was constructed in [39].

When the Lie 3-algebra does not have a central element, the fermionic shift symmetry is absent. In this case the space-time supersymmetry should be less compared with the flat space. This may be regarded as a supersymmetric counterpart of the absence of space-time translational symmetry in the orbifold interpretation of the model based on so-called $\mathcal{A}_4$ algebra [28 – 30].

The BLG model is superconformal at the classical level, and expected to be so at the quantum level. The superconformal symmetry should correspond to the near horizon super-isometry in AdS-CFT correspondence, and this is one of the strongest motivations for studying this model. It will be interesting to construct central extension of the superconformal algebra explicitly in the BLG model.

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Appendix

A. Notation for indices

$$\begin{aligned}
\text{worldvolume coordinates} &: \mu, \nu = 0, 1, 2 \\
\text{spatial worldvolume coordinates} &: i, j = 1, 2 \\
\text{transverse space coordinates} &: I, J = 3, \cdots, 10 \\
\text{all 11D coordinates} &: m, n = 0, 1, \cdots, 10 \\
\text{Spin}(1,10) \text{ spinor indices} &: \alpha, \beta = 1, \cdots, 32 \\
\text{basis of Lie 3-algebra } \mathcal{A} &: a, b, \cdots, \dim \mathcal{A}
\end{aligned} \tag{A.1}$$

B. Eleven dimensional Clifford algebra

11D Gamma matrices

$$\{\Gamma^m, \Gamma^n\} = 2\eta^{mn} \quad (m, n = 0, 1, \cdots, 10). \tag{B.1}$$

We use mostly plus convention, i.e. $\eta_{00} = -1$, $\eta_{mn} = \delta_{mn} (m, n \neq 0)$. The charge conjugation matrix C in eleven dimension satisfies

$$C^{-1}\Gamma^m C = -(\Gamma^m)^T, \quad C^T = -C, \quad C^\dagger C = 1. \tag{B.2}$$

$$\begin{aligned}
\Gamma^{m_1 m_2 \cdots m_r} &\equiv \frac{1}{r!}\Gamma^{[m_1} \cdots \Gamma^{m_r]} \\
&= \Gamma^{m_1}\Gamma^{m_2} \cdots \Gamma^{m_r} \quad (\text{when all } m_s \text{ are different}). \\
&= 0 \quad (\text{otherwise}).
\end{aligned} \tag{B.3}$$

$\Gamma^{m_1 m_2 \cdots m_r} C$ is a symmetric matrix for $r = 1, 2, 5, 6, 9, 10$.

B.1 $\mathrm{Spin}(1,2) \otimes \mathrm{Spin}(8)$ decomposition

We define

$$\Gamma \equiv \Gamma_{012} = \frac{1}{3!}\varepsilon^{\mu\nu\rho}\Gamma_\mu \Gamma_\nu \Gamma_\rho, \tag{B.4}$$

where $\varepsilon^{\mu\nu\rho}$ is the totally anti-symmetric tensor with $\varepsilon^{012} = 1$.

$$[\Gamma, \Gamma^\mu] = 0, \quad \{\Gamma, \Gamma^I\} = 0. \tag{B.5}$$

$$C\Gamma^T = \Gamma C. \tag{B.6}$$

$$\Gamma_\pm \equiv \frac{1 \pm \Gamma}{2}. \tag{B.7}$$

Decomposition:

$$\Gamma^\mu = \gamma^\mu \otimes \bar{\gamma}_9, \quad \Gamma^I = 1 \otimes \bar{\gamma}^I, \tag{B.8}$$

where γ^μ's are gamma matrices in (1+2)D and $\bar{\gamma}$'s are that of 8D, and

$$\bar{\gamma}_9 \equiv \bar{\gamma}^3 \cdots \bar{\gamma}^{10}. \tag{B.9}$$

If we choose the basis for the (1+2)D Clifford algebra as

$$\gamma_0 = i\sigma_2, \quad \gamma_1 = \sigma_1, \quad \gamma_2 = \sigma_3, \tag{B.10}$$

then

$$\Gamma \equiv \Gamma_{012} = 1 \otimes \bar{\gamma}_9, \tag{B.11}$$

i.e. the chirality for Γ and $\bar{\gamma}_9$ becomes the same.

C. Majorana spinors

Majorana condition in 11D:

$$\Psi = C\bar{\Psi}^T. \tag{C.1}$$

Conjugate momentum (for kinetic terms the same to (2.8))

$$\Pi_{\Psi_\alpha} = \frac{i}{2}(\bar{\Psi}\Gamma^0)_\alpha = \frac{i}{2}(\Gamma^{0T}C^{-1}\Psi)_\alpha. \tag{C.2}$$

Dirac bracket:

$$\{\Psi_\alpha, \Psi_\beta\}_D = -i\left(\Gamma_-\Gamma^0 C\right)_{\alpha\beta}, \tag{C.3}$$

where here and in the following we suppress space coordinates and spinor indices when it is obvious.

D. Supercharge commutation relations

$$q^{NL} = -\Gamma^0\Psi_\odot, \tag{D.1}$$

$$\begin{aligned} q^L = & -\langle\Gamma^I\Psi, D_0X^I\rangle - \langle\Gamma^I\Gamma^0\Gamma^i\Psi, D_iX^I\rangle \\ & -\frac{1}{6}\langle\Gamma^{IJK}\Gamma^0\Psi, [X^I, X^J, X^K]\rangle. \end{aligned} \tag{D.2}$$

For any fields Φ in the BLG model,

$$i\{\bar{\eta}_\alpha Q_\alpha^{NL}, \Phi\}_D = \delta_\eta\Phi, \tag{D.3}$$

$$i\{\bar{\epsilon}_\alpha Q_\alpha, \Phi\}_D = \delta_\epsilon\Phi. \tag{D.4}$$

We obtain

$$i\{q_\alpha^{NL}, \Psi_{\odot\beta}\}_D = (\Gamma_- C)_{\alpha\beta}. \tag{D.5}$$

Dirac brackets for super charges:

$$
\begin{aligned}
i\{q^{NL}_\alpha, Q^{NL}_\beta\}_D &= \left(\Gamma_-\Gamma^0 C\right)_{\alpha\beta}, && \text{(D.6)}\\
i\{q^{NL}, Q^{L}\}_D + i\{q^{L}, Q^{NL}\}_D &= \Gamma^I C\partial_0 X^I_\odot - \varepsilon^{0i}{}_j\Gamma^j\Gamma^I C\partial_i X^I_\odot \\
&\quad -\frac{1}{12}\varepsilon^0{}_{ij}\langle [X^I, X^J, X^K], T_\odot\rangle \Gamma^{ijIJK}C. && \text{(D.7)}
\end{aligned}
$$

This leads to (2.37).

$$
\begin{aligned}
i\{q^L, Q^L\}_D &= 2p_\mu \Gamma_+\Gamma^\mu C \\
&\quad -\langle D_i X^I, D_j X^J\rangle \varepsilon^{0ij}\Gamma_+\Gamma^{IJ}C \\
&\quad +\langle D_0 X^I, [X^I, X^J, X^K]\rangle \Gamma_+\Gamma^{JK}C \\
&\quad +\frac{1}{3}\langle D_i X^I, [X^J, X^K, X^L]\rangle \Gamma_+\Gamma^{0iIJKL}C \\
&\quad -\frac{1}{4}\langle [X^I, X^J, X^K], [X^I, X^L, X^M]\rangle \Gamma_+\Gamma^{0JKLM}C. \qquad \text{(D.8)}
\end{aligned}
$$

This leads to (2.18).

The bosonic part of the energy-momentum tensor:

$$
T_{\mu\nu} = \langle D_\mu X^I, D_\nu X^I\rangle - \eta_{\mu\nu}\left(\frac{1}{2}\langle D^\rho X^I, D_\rho X^I\rangle + V(X)\right), \qquad \text{(D.9)}
$$

The bosonic part of the Hamiltonian density:

$$
\mathcal{H} = \frac{1}{2}\langle P^I, P^I\rangle + \sum_{i=1,2}\frac{1}{2}\langle D_i X^I, D_i X^I\rangle + V(X). \qquad \text{(D.10)}
$$

The momentum densities:

$$
p_0 = \mathcal{H}, \quad p_i = \langle D_0 X^I, D_i X^I\rangle, \quad p_I = \partial_0 X^I_\odot. \qquad \text{(D.11)}
$$

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